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Determining Dielectric Constant and Loss Tangent in FR-4

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1. Introduction

The design of high-speed circuits requires a knowledge of the dielectric constant (ε_r) and loss parameter ($tan\delta$) of the substrate. This is particularly important when using FR-4 material because of the wide variations encountered among different vendors and even between different orders from the same vendor. It is therefore desirable to develop a method allowing the design engineer to personally determine the ε_r and $tan\delta$ of a substrate material.

2. Measurement Procedure

The procedure begins by metalizing the substrate on all sides hence forming a rectangular resonant cavity. A small hole is drilled through the material for insertion of the probe feed. Next, the S_{11} parameter is obtained experimentally using a network analyzer. The dimension of the rectangular cavity is chosen such that the first dip in $|S_{11}|$ versus frequency corresponds to the resonant frequency and quality factor (Q) of the TE_{101} mode within the cavity. Here, the second mode index refers to the lack of variation over the thickness of the substrate.

3. Determining Dielectric Constant ε_r

The theoretical resonant frequency for TE_{101} mode is given by

$$f = \frac{c}{2\pi\sqrt{\mu_r \varepsilon_r}}\sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{d}\right)^2} \tag{1}$$

where μ_r is relative permeability of the substrate. For FR-4, μ_r equals 1; c is the speed of light in free space; while a and d are the length and width of the substrate respectively. Thus, if the resonant frequency f is known, ε_r can be obtained by solving equation (1). As indicated in [1], if extremely accurate results are desired, the following correction can be made

$$f_0 = f_m \Big/ \left(1 - \frac{1}{2Q}\right) \tag{2}$$

where f_m is the measured frequency and f_0 is the corrected frequency that should be used in (1) above.

4. Determining Loss Tangent *tanδ*

4.1. Determing Unloaded Quality Factor (Qu)

A generalized loaded Q is introduced as $Q_L(x)$ in [2].

$$Q_L(x) = \frac{\omega_0}{(\Delta\omega)_x} \tag{3}$$

where ω_0 is the resonant angle frequency and $(\Delta\omega)_x$ is the bandwidth measured between points where $|S_{11}|=x$. The unloaded Q can be expressed as [2]

$$Q_u = Q_L(x)F(x) \tag{4}$$

where

$$F(x) = \frac{2}{1 \pm |S_{11}|_0} \sqrt{\frac{|S_{11}|_x^2 - |S_{11}|_0^2}{1 - |S_{11}|_x^2}} \tag{5}$$

where $|S_{11}|_0$ is the magnitude of S_{11} at ω_0. $|S_{11}|_x$ represents the magnitude of S_{11} that is equal to x. The "-" and "+" signs in (5) correspond to the over-coupling and under-coupling respectively. These two cases can be easily distinguished by inspecting S_{11} response in Smith Chart from a network analyzer (such as HP8753D). Basically, a large circle enclosing the origin of the Smith Chart signifies an over-coupled case. For under-coupling, the S_{11} circle is small and excludes the origin.

4.2. Determing Loss Tangent (tanδ)

The unloaded quality factor Q_u obtained above consists of Q_d due to dielectric loss and Q_c due to conducting walls loss.

$$\frac{1}{Q_u} = \frac{1}{Q_d} + \frac{1}{Q_c} \tag{6}$$

For TE_{101} mode, Q_c of the rectangular cavity with lossy walls and a lossless dielectric can be found analytically as [3]

$$Q_c = \frac{(kad)^3 b\eta}{2\pi^2 R_s (2a^3 b + 2bd^3 + a^3 d + ad^3)} \tag{7}$$

where $k = \omega\sqrt{\mu\varepsilon}$ is the wavenumber in the substrate and $\eta = \sqrt{\frac{\mu}{\varepsilon}}$ is the wave impedance;

$R_s = \sqrt{\frac{\omega\mu}{\sigma}}$ is the surface resistivity of the metalic walls; and Q_d is related to loss tangent by [3]

$$\tan\delta = \frac{1}{Q_d} \qquad (8)$$

5. Measurement Results

A double-sided FR-4 board with dimensions of 9.1-cm by 40-mils by 10.8 cm is sealed by copper tape, forming a rectangular resonant cavity. S_{11} measurements were performed by using the HP 8753D network analyzer. The dielectric constant and the loss tangent are determined using the procedure described in previous section. The results are shown in Table 1.

Table 1: ε_r and $tan\delta$ results

f_0 (GHz)	ε_r	$tan\delta$	Effective conductivity at f_0 (S/m)
1.0185	4.47	0.01646	0.004168

Next, FDTD modeling was performed using the parameters in Table 1. The S11 parameter obtained from FDTD modeling is compared with that from the measurement, as shown in Figure 1. The red curve, corresponding to FDTD modeling, agrees with the resonant frequency of the green curve, corresponding to the measurement, within 1%. The dip in the red curve agrees with that of green curve within 3-dB.

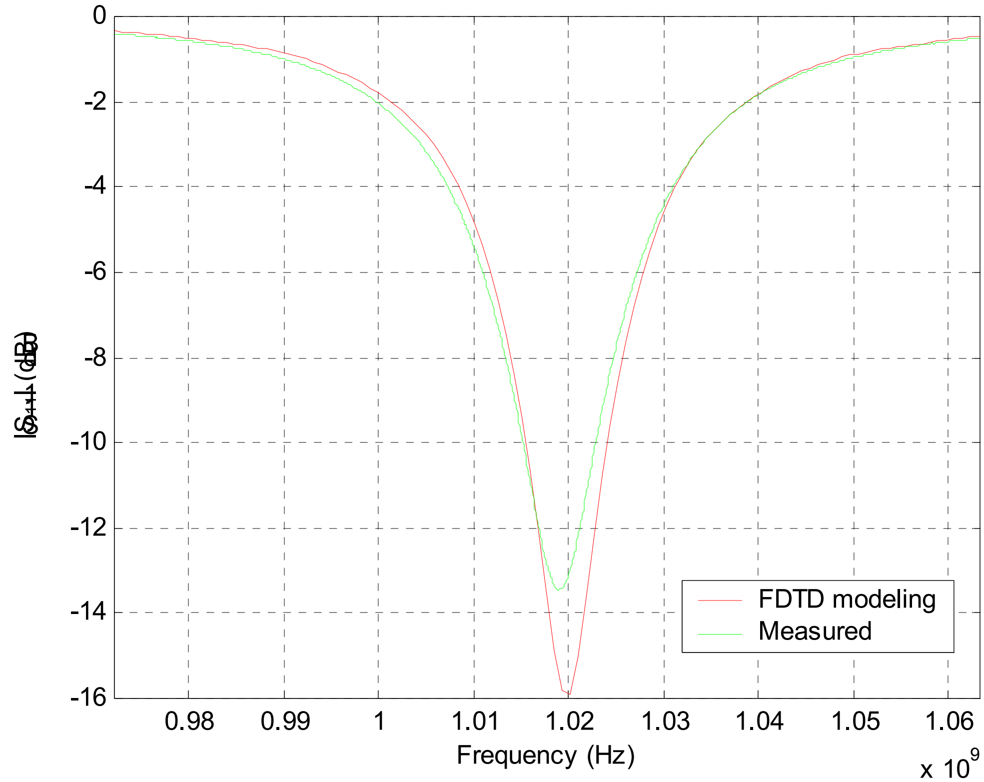

Figure 1. Measured $|S_{11}|$ compared with $|S_{11}|$ from FDTD modeling using measured ε_r and $tan\delta$.

6. Summary

A simple technique for determining dielectric constant and loss tangent of a substrate is presented. This technique allows the design engineer to measure the ε_r and $tan\delta$. It is very useful when applied to FR-4 material because of the wide variations of ε_r and $tan\delta$ observed.

7. References

[1] John Q. Howell, "A Quick Accurate Method to Measure the Dielectric Constant of a Microwave Integrated-Circuit Substrate", *IEEE Trans. on Microwave Theory and Technique,* vol. MTT-21, pp. 142-143, March 1973.

[2] Raymond S. Kwok and Ji-Fuh Liang, "Characterization of High-Q Resonator for Microwave-Filter Application", *IEEE Trans. on Microwave Theory and Techniques,* vol. 47, No. 1, January 1999.

[3] David M. Pozar, *Microwave Engineering*, Second Edition, John Wiley & Sons, Inc., 1998.